State estimation is studied for a special class of flag Hidden Markov Models (HMMs), which comprise 1) an arbitrary finite-state underlying Markov chain and 2) a structured observation process wherein a subset of states emit distinct flags with some probability while other states are unmeasured. For flag HMMs, an explicit computation of the probability of error for the maximum-likelihood filter and smoother is developed. Also, the form of the optimal filter is further characterized in terms of the time since the last flag, and this result is used to further simplify the error-probability computation. Some preliminary graph-theoretic insights into the error probability and its computation are discussed. Finally, these algebraic and structural results are leveraged to address sensor placement in two examples, including one on activity-monitoring in a home environment that is drawn from field data. These examples indicate that low error-probability filtering and smoothing can be achieved with relatively few sensors.