A reliable support detection is essential for a greedy algorithm to reconstruct a sparse signal accurately from compressed and noisy measurements. This paper proposes a novel support detection method for greedy algorithms, which is referred to as maximum a posteriori (MAP) support detection. Unlike existing support detection methods that identify support indices with the largest correlation value in magnitude per iteration, the proposed method selects them with the largest likelihood ratios computed under the true and null support hypotheses by simultaneously exploiting the distributions of a sensing matrix, a sparse signal, and noise. Leveraging this technique, MAP-Matching Pursuit (MAP-MP) is first presented to show the advantages of exploiting the proposed support detection method, and a sufficient condition for perfect signal recovery is derived for the case when the sparse signal is binary. Subsequently, a set of iterative greedy algorithms, called MAP-generalized Orthogonal Matching Pursuit (MAP-gOMP), MAP-Compressive Sampling Matching Pursuit (MAP-CoSaMP), and MAP-Subspace Pursuit (MAP-SP) are presented to demonstrate the applicability of the proposed support detection method to existing greedy algorithms. From empirical results, it is shown that the proposed greedy algorithms with highly reliable support detection can be better, faster, and easier to implement than basis pursuit via linear programming.