Several popular period estimation techniques use union-of-subspaces models to represent periodic signals. The main idea behind these techniques is to compare the components of the signal along a set of subspaces representing different periods. Such techniques were shown to offer important advantages over traditional methods, such as those based on DFT. So far, most of these subspace techniques have been developed independent of each other, and there has not been a unified theory analyzing them from a common perspective. In this paper, all such methods are first unified under one general framework. Further, several fundamental aspects of such subspaces are investigated, such as the conditions under which a generic set of subspaces offers unique periodic decompositions, their minimum required dimensions, etc. A number of basic questions in the context of dictionaries spanning periodic signals are also answered. For example, what is the theoretically minimum number of atoms required in any type of dictionary, in order to be able to represent periods $1 \leq P \leq P_{max}$? For each period P , what should be the minimum dimension of the subspace of atoms representing the Pth period itself? Unlike in traditional Fourier dictionaries, it is shown that nonuniform and compact grids are crucial for period estimation. Interestingly, it will be seen that the Euler totient function from number theory plays an important role in providing the answers to all such questions.