

The Takens-Bogdanov bifurcation is a codimension-two bifurcation that provides a key to the presence of complex dynamics in many systems of physical interest. When the system is translation-invariant in one spatial dimension with no left-right preference the imposition of periodic boundary conditions leads to the Takens-Bogdanov bifurcation with $O(2)$ symmetry. This bifurcation, analyzed by G. Dangelmayr and E. Knobloch, *Phil. Trans. R. Soc. London A* 322, 243 (1987), describes the interaction between steady states and travelling and standing waves in the nonlinear regime and predicts the presence of modulated travelling waves as well. The analysis reveals the presence of several global bifurcations near which the averaging method (used in the original analysis) fails. We show here, using a combination of numerical continuation and the construction of appropriate return maps, that near the global bifurcation that terminates the branch of modulated travelling waves, the normal form for the Takens-Bogdanov bifurcation admits cascades of period-doubling bifurcations as well as chaotic dynamics of Shil'nikov type. Thus chaos is present arbitrarily close to the codimension-two point.

