

We consider N oscillators coupled by a mean field as in the Winfree model. The model is governed by two parameters: the coupling strength ϵ and the spectrum width σ of the frequencies of each oscillator centred at 1. In the uncoupled regime, $\epsilon = 0$, each oscillator possesses its own natural frequency, and the difference between the phases of any two oscillators grows linearly in time. In the zero-width regime for the spectrum, the oscillators are simultaneously in the death state if and only if ϵ is above some positive value ϵ^* . We say that N oscillators are synchronized if the difference between any two phases is uniformly bounded in time. We identify a new hypothesis for the existence of synchronization. The domain in (ϵ, σ) of synchronization contains $\{0\} \times [0, \epsilon^*]$ in its closure. Moreover, the domain is independent of the number of oscillators and the distribution of the frequencies. We show numerically, on a specific family of Winfree models, that the above hypothesis seems to be a bifurcation criterion for the existence of synchronization domain. The transition is not, however, mathematically sharp.

