Consider networks in which all arrows are distinct and all cells are distinct. In this context, we obtain complete descriptions of the groups of diffeomorphisms that preserve network dynamics in the following sense: changing coordinates via the diffeomorphism transforms the space of admissible maps to itself. Five distinct actions are considered: left, right, contact, conjugacy, and vector field. Key features are the left core, right core, and core of the network. The core is a subnetwork with special combinatorial features, and it represents a partition of the cells into all-to-all connected subnetworks that couple to each other in a feed-forward manner. For the left/right actions, the group consists of all diffeomorphisms that are admissible for the left/right core, respectively. The contact action is a pair (B, ?) where B is determined by the left core and ? by the right core. For the conjugacy and vector field actions, the group is generated by diffeomorphisms that are admissible for the core together with graph automorphisms of the network; that is, permutations of the cells that map arrows to arrows but need not preserve arrow type. The proofs are combinatorial for the left and right actions, but require a mixture of Lie theory and the structure theory of associative algebras in the other cases, together with a nonlinear-to-linear reduction theorem.