The coupled cell formalism is a systematic way to represent and study coupled nonlinear differential equations using directed graphs. In this work, we focus on coupled cell systems in which individual cells are also Hamiltonian. We show that some coupled cell systems do not admit Hamiltonian vector fields because the associated directed graphs are incompatible. In broad terms, we prove that only systems with bidirectionally coupled digraphs can be Hamiltonian. Aside from the topological criteria, we also study the linear theory of regular Hamiltonian coupled cell systems, i.e. systems with only one type of node and one type of coupling. We show that the eigenspace at a codimension-one bifurcation from a synchronous equilibrium of a regular Hamiltonian network can be expressed in terms of the eigenspaces of the adjacency matrix of the associated directed graph. We then prove results on steady-state bifurcations and a version of the Hamiltonian Hopf theorem.