

We present a method of combining coupled cell systems to get dynamics supporting robust simple heteroclinic networks given by the product of robust simple heteroclinic networks (cycles). We consider coupled cell networks, with no assumption on symmetry, and combine them via the join operation. Assuming that the dynamics of the component networks supports robust simple heteroclinic cycles or networks, we show that the join dynamics realizes a more complex heteroclinic network given by the product of those cycles or networks. Moreover, the equilibria in the product heteroclinic network correspond to partially synchronous states. Assuming no symmetry for the component coupled cell networks, one of the key points for the existence and robustness of the heteroclinic dynamics is the flow-invariant subspaces forced by the network structure – the synchrony subspaces. The other key point is that the (linear) stability of equilibria in the join dynamics is determined by the (linear) stability of equilibria in the component dynamics. The first point depends only at the network structure of the component networks. The second one depends both at the components network structures and the convenient choice of the join coupling. The proposed method is general and can be applied to the join of symmetric or asymmetric networks. Here, we illustrate it through the join of two asymmetric coupled cell networks where robust simple heteroclinic cycles between fully synchronous equilibria occur. We obtain robust simple heteroclinic networks for the join dynamics between partially synchronized equilibria for the associated join network.

