In this paper, we study the control of a class of time-invariant linear ensemble systems whose natural dynamics are linear in the system parameter. This class of ensemble control systems arises from practical engineering and physical applications, such as transport of quantum particles and control of uncertain harmonic systems. We establish explicit algebraic criteria to examine controllability of such ensemble systems. Our derivation is based on the notion of polynomial approximation, where the elements of the reachable set of the ensemble system are represented in polynomials of the system parameter and used to approximate the desired state of interest. In addition, we highlight the role of the spectra of the system matrices play in the determination of ensemble controllability. Finally, illustrative examples and numerical simulations for optimal control of this class of linear ensemble systems are presented to demonstrate the theoretical results.