We propose a novel operator-theoretic framework to study global stability of nonlinear systems. Based on the spectral properties of the so-called Koopman operator, our approach can be regarded as a natural extension of classic linear stability analysis to nonlinear systems. The main results establish the (necessary and sufficient) relationship between the existence of specific eigenfunctions of the Koopman operator and the global stability property of hyperbolic fixed points and limit cycles. These results are complemented with numerical methods which are used to estimate the region of attraction of the fixed point or to prove in a systematic way global stability of the attractor within a given region of the state space.