We investigate how the following properties are related to each other: i) A manifold is “transversally” exponentially stable; ii) the “transverse” linearization along any solution in the manifold is exponentially stable; and iii) there exists a field of positive definite quadratic forms whose restrictions to the directions transversal to the manifold are decreasing along the flow. We illustrate their relevance with the study of exponential incremental stability. Finally, we apply these results to two control design problems, nonlinear observer design and synchronization. In particular, we provide necessary and sufficient conditions for the design of nonlinear observer and of nonlinear synchronizer with exponential convergence property.