We consider two one-parameter families of piecewise isometries of a rhombus. The rotational component is fixed, and its coefficients belong to a quadratic number field $\mathbb{Q}(\sqrt{d})$. The translations depend on a parameter *s* which is allowed to vary in an interval. We investigate renormalizability, and show that recursive constructions of first-return maps on a suitable subdomain eventually produce a scaled-down replica of this domain, but with a renormalized parameter *r*(*s*). We treat two quadratic fields: *d* = 5, 2. In the first case, the renormalization map *r* is of Lüroth type (a piecewise-affine version of Gauss' map), whereas in the second case, it is the second iterate of a map *f* of this type. We show that exact self-similarity corresponds to the eventually periodic points of *r* (resp. *f*), and that such parameter values are precisely the elements of the quadratic field that lie in the given interval. The renormalizability proof for $\mathbb{Q}(\sqrt{5})$ is based on a straightforward application of return-map analysis. The octagonal case $\mathbb{Q}(\sqrt{2})$ is far more challenging. The proof is organized by a graph analogous to those used to construct renormalizable interval-exchange transformations. There are 10 distinct renormalization scenarios corresponding to as many closed circuits in the graph. The process of induction along some of these circuits involves intermediate maps undergoing, as the parameter varies, infinitely many bifurcations. Our proofs rely on computer assistance.