For the problem of multi class linear classification and feature selection, we propose approximate message passing approaches to sparse multinomial logistic regression (MLR). First, we propose two algorithms based on the Hybrid Generalized Approximate Message Passing framework: one finds the maximum a posteriori linear classifier and the other finds an approximation of the test-error-rate minimizing linear classifier. Then we design computationally simplified variants of these two algorithms. Next, we detail methods to tune the hyperparameters of their assumed statistical models using Stein's unbiased risk estimate and expectation-maximization, respectively. Finally, using both synthetic and real-world datasets, we demonstrate improved error-rate and runtime performance relative to existing state-of-the-art approaches to sparse MLR.