Next-generation communication systems have to comply with very strict requirements for increased flexibility in heterogeneous environments, high spectral efficiency, and agility of carrier aggregation. This fact motivates research in advanced multicarrier modulation (MCM) schemes, such as filter bank-based multicarrier (FBMC) modulation. This paper focuses on the offset quadrature amplitude modulation (OQAM)-based FBMC variant, known as FBMC/OQAM, which presents outstanding spectral efficiency and confinement in a number of channels and applications. Its special nature, however, generates a number of new signal processing challenges that are not present in other MCM schemes, notably, in orthogonal-frequency-division multiplexing (OFDM). In multiple-input multiple-output (MIMO) architectures, which are expected to play a primary role in future communication systems, these challenges are intensified, creating new interesting research problems and calling for new ideas and methods that are adapted to the particularities of the MIMO-FBMC/OQAM system. The goal of this paper is to focus on these signal processing problems and provide a concise yet comprehensive overview of the recent advances in this area. Open problems and associated directions for future research are also discussed.