This paper considers the design of robust filters for radar pulse-Doppler processing when the interference is a wide sense stationary random process. The figure of merit which is optimized is the signal-to-interference-plus-noise ratio (SINR) at the filter output under a multitude of constraints accounting for Doppler filter sidelobes as well as uncertainties both in the received useful signal component and interference covariance matrix. The design is analytically formulated as a constrained optimization problem whose solvability is thoroughly studied. Precisely, a polynomial-time solution technique to get the optimal filter is proposed exploiting the representation of non-negative trigonometric polynomials via linear matrix inequalities, the spectral factorization theorem, and the duality theory. Last but not least, a detailed analysis of the optimum filter performance is provided showing the tradeoffs involved in the design and the gain achievable over some already known counterparts.