We consider a downlink multiuser MISO system with bounded errors in the channel state information at the transmitter (CSIT). We first look at the robust design problem of achieving max-min fairness amongst users (in the worst-case sense). Contrary to the conventional approach adopted in literature, we propose a rather unorthodox design based on a rate-splitting (RS) strategy. Each user's message is split into two parts, a common part and a private part. All common parts are packed into one super common message encoded using a public codebook, while private parts are independently encoded. The resulting symbol streams are linearly precoded and simultaneously transmitted, and each receiver retrieves its intended message by decoding both the common stream and its corresponding private stream. For CSIT uncertainty regions that scale with SNR (e.g., by scaling the number of feedback bits), we prove that a RS-based design achieves higher max-min (symmetric) degrees of freedom (DoF) compared with conventional designs (NoRS). For the special case of nonscaling CSIT (e.g., fixed number of feedback bits), and contrary to NoRS, RS can achieve a nonsaturating max-min rate. We propose a robust algorithm based on the cutting-set method coupled with the weighted minimum mean-square error (WMMSE) approach, and we demonstrate its performance gains over state-of-the-art designs. Finally, we extend the RS strategy to address the quality of service (QoS) constrained power minimization problem, and we demonstrate significant gains over NoRS-based designs.