The linear canonical transform (LCT), which generalizes many classical transforms, has been shown to be a powerful tool for signal processing and optics. Sampling theory of the LCT for bandlimited signals has blossomed in recent years. However, in practice signals are never perfectly bandlimited, and in many cases measurement devices are nonideal. The objective of this paper is to develop a sampling theorem for the LCT from general measurements, which can provide a suitable and realistic model of sampling and approximation for real-world applications. We first describe a general class of approximation spaces for the LCT and provide a full characterization of their basis functions. Then, we propose a generalized sampling theorem for arbitrary measurement and approximation spaces associated with the LCT. Several properties of the proposed sampling theorem are also discussed. Furthermore, the approximation error is estimated. Finally, numerical results and several applications of the derived results are presented.