The authors develop a dissipativity theory for switched systems whose dynamical modes are described by systems of higher order linear differential equations. They give necessary and sufficient conditions for dissipativity based on systems of linear matrix inequalities (LMIs), constructed from the coefficient matrices of the differential equations describing the modes. The relationship between dissipativity and stability is also discussed, and an application to the stabilization of power converters is provided.