In this paper, a novel approach is introduced for the solution of the non-linear Troesch's boundary value problem. The underlying strategy is based on Green's functions and fixed-point iterations, including Picard's and Krasnoselskii–Mann's schemes. The resulting numerical solutions are compared with both the analytical solutions and numerical solutions that exist in the literature. Convergence of the iterative schemes is proved via manipulation of the contraction principle. It is observed that the method handles the boundary layer very efficiently, reduces lengthy calculations, provides rapid convergence, and yields accurate results particularly for large eigenvalues. Indeed, to our knowledge, this is the first time that this problem is solved successfully for very large eigenvalues, actually the rate of convergence increases as the magnitude of the eigenvalues increases.