In this paper, a work performed to allow massively parallel finite element flow computations is presented. It includes the development and optimisation of two particular features of a finite element multiphase computational fluid dynamics software, which are mesh generation and linear system solution, using anisotropic adaptation and multigrid preconditioning. Parallel performances on supercomputers are shown, where the largest generated mesh (on 65 536 Intel Xeon or 261 144 Power PC cores) had 33.4 billions of nodes, leading to a 100 billion of unknowns linear system solution. Final applications concern, between others, image-based flow simulations.