This paper examines stochastic optimal control problems in which the state is perfectly known, but the controller's measure of time is a stochastic process derived from a strictly increasing Lévy process. We provide dynamic programming results for continuous-time finite-horizon control and specialize these results to solve a noisy-time variant of the linear quadratic regulator problem and a portfolio optimization problem with random trade activity rates. For the linear quadratic case, the optimal controller is linear and can be computed from a generalization of the classical Riccati differential equation.