The paper introduces and studies differentially positive systems, that is, systems whose linearization along an arbitrary trajectory is positive. A generalization of Perron-Frobenius theory is developed in this differential framework to show that the property induces a conal order that strongly constrains the asymptotic behavior of solutions. The results illustrate that behaviors constrained by local order properties extend beyond the well-studied class of linear positive systems and monotone systems, which both require a constant cone field and a linear state space.