Classical theorems on the complete characterization of stability boundaries provided little information about the existence and types of unstable equilibrium points on a stability boundary. In this technical note, we derive analytical results and sufficient conditions about the existence of all types of unstable equilibrium points in a dynamical system whose stability regions are uniformly bounded. Some applications of these analytical results are illustrated using electric power systems.