The aim of this paper is to propose a new numerical approximation of the Kalman-Bucy filter for semi-Markov jump linear systems. This approximation is based on the selection of typical trajectories of the driving semi-Markov chain of the process by using an optimal quantization technique. The main advantage of this approach is that it makes pre-computations possible. We derive a Lipschitz property for the solution of the Riccati equation and a general result on the convergence of perturbed solutions of semi-Markov switching Riccati equations when the perturbation comes from the driving semi-Markov chain. Based on these results, we prove the convergence of our approximation scheme in a general infinite countable state space framework and derive an error bound in terms of the quantization error and time discretization step. We employ the proposed filter in a magnetic levitation example with markovian failures and compare its performance with both the Kalman-Bucy filter and the Markovian linear minimum mean squares estimator.