The maximum power and power ramp rate are important grid codes for integrating renewable energy resources in transmission systems. The power curtailment regulates the maximum power and ramp rate; however, adding an energy storage system (ESS) can time shift surplus wind energy instead of curtailing it. The flywheel energy storage system (FESS) has the advantages of high efficiency and long lifetime; however, it has non-negligible standby losses and its lifetime is reduced exponentially as the rotating speed increases. Considering such practical constraints, this work presents an energy management system (EMS) for a hybrid power system composed of a wind farm with a FESS. The FESS time shifts the surplus wind energy to respect the grid codes and reduce wind curtailment; meanwhile, the EMS aims at minimizing the FESS standby losses and boosting its lifetime using the predicted wind power data. The EMS is composed of two controllers. The first controller is a linear model predictive controller that defines the long-term FESS power set-point. The second controller is a real-time adaptive hysteresis controller that compensates for the wind-power prediction error. Comparative simulation studies and hardware-in-the-loop test results validate the effectiveness of the proposed EMS in reducing the FESS losses while respecting the grid integration code constraints.