Analysis of transient stability of strongly nonlinear post-fault dynamics is one of the most computationally challenging parts of dynamic security assessment. This paper proposes a novel approach for assessment of transient stability of the system. The approach generalizes the idea of energy methods, and extends the concept of energy function to a more general Lyapunov functions family (LFF) constructed via semidefinite programming techniques. Unlike the traditional energy function and its variations, the constructed Lyapunov functions are proven to be decreasing only in a finite neighborhood of the equilibrium point. However, we show that they can still certify stability of a broader set of initial conditions in comparison to the energy function in the closest-UEP method. Moreover, the certificates of stability can be constructed via a sequence of convex optimization problems that are tractable even for large scale systems. We also propose specific algorithms for adaptation of the Lyapunov functions to specific initial conditions and demonstrate the effectiveness of the approach on a number of IEEE test cases.