We study the problem of detecting transmission line outages in power grids. We model the time series of power network measurements as a hidden Markov process, and formulate the line outage detection problem as an inference problem. Due to the physical nature of the line failure dynamics, the transition probabilities of the hidden Markov Model are sparse. Taking advantage of this fact, we further propose an approximate inference algorithm using particle filtering, which takes in the times series of power network measurements and produces a probabilistic estimation of the status of the transmission line status. We then assess the performance of the proposed algorithm with case studies. We show that it outperforms the conventional static line outage detection algorithms and is robust to both measurement noise and model parameter errors.