This paper investigates the use of a qualitative inverse scattering method known as the linear sampling method (LSM) for imaging underwater scenes using limited aperture receiver configurations. The LSM is based on solving a set of unstable integral equations known as the far-field equations and whose stability breaks down even further for under-sampled observation aperture data. Based on the results of a recent study concerning multi-frequency LSM imaging, we propose an iterative inversion method that is founded upon a compressive sensing framework. In particular, we leverage multi-frequency diversity in the data by imposing a partial frequency variation prior on the solution which we show is justified when the frequency bandwidth is sampled finely enough. We formulate an alternating direction method of multiplier approach to minimize the proposed cost function. Proof of concept is established through numerically generated data as well as experimental acoustic measurements taken in a shallow pool facility at the U.S Naval Research Laboratory.