We propose a sampling scheme that can perfectly reconstruct a collection of spikes on the sphere from samples of their lowpass-filtered observations. Central to our algorithm is a generalization of the annihilating filter method, a tool widely used in array signal processing and finite-rate-of-innovation (FRI) sampling. The proposed algorithm can reconstruct K spikes from $(K+\sqrt{K})^2$ spatial samples. For large K, this sampling requirement improves over previously known FRI sampling schemes on the sphere by a factor of four. We showcase the versatility of the proposed algorithm by applying it to three problems: 1) sampling diffusion processes induced by localized sources on the sphere, 2) shot noise removal, and 3) sound source localization (SSL) by a spherical microphone array. In particular, we show how SSL can be reformulated as a spherical sparse sampling problem.