Compute-and-forward (CF) harnesses interference in a wireless network by allowing relays to compute combinations of source messages. The computed message combinations at relays are correlated, and so directly forwarding these combinations to a destination generally incurs information redundancy and spectrum inefficiency. To address this issue, we propose a novel relay strategy, termed compute-compress-and-forward (CCF). In CCF, source messages are encoded using nested lattice codes constructed on a chain of nested coding and shaping lattices. A key difference of CCF from CF is an extra compressing stage inserted in between the computing and forwarding stages of a relay, so as to reduce the forwarding information rate of the relay. The compressing stage at each relay consists of two operations: first to quantize the computed message combination on an appropriately chosen lattice (referred to as a quantization lattice), and then to take modulo on another lattice (referred to as a modulo lattice). We study the design of the quantization and modulo lattices and propose successive recovering algorithms to ensure the recoverability of source messages at destination. Based on that, we formulate a sum-rate maximization problem that is in general an NP-hard mixed integer program. A low-complexity algorithm is proposed to give a suboptimal solution. Numerical results are presented to demonstrate the superiority of CCF over the existing CF schemes.