Sparse data models are powerful tools for solving ill-posed inverse problems. We present a regularization framework based on the sparse synthesis and sparse analysis models for problems governed by linear partial differential equations. Although nominally equivalent, we show that the two models differ substantially from a computational perspective: unlike the sparse synthesis model, its analysis counterpart has much better scaling capabilities and can indeed be faster when more measurement data is available. Our findings are illustrated on two examples, sound source localization and brain source localization, which also serve as showcases for the regularization framework. To address this type of inverse problems, we develop a specially tailored convex optimization algorithm based on the Alternating Direction Method of Multipliers.