This paper presents a formal definition of stability for node centrality measures in weighted graphs. It is shown that the commonly used measures of degree, closeness and eigenvector centrality are stable, whereas betweenness centrality is not. An alternative definition of the latter that preserves the same centrality notion while satisfying the stability criteria is introduced. Continuity is presented as a less stringent alternative to stability. Betweenness centrality is shown to be not only unstable but discontinuous. Numerical experiments in synthetic random networks and real-world data show that, in practice, stability and continuity imply different levels of robustness in the presence of noisy data. In particular, the stable betweenness centrality is shown to exhibit resilience against noise that is absent in the discontinuous and unstable standard betweenness centrality, while preserving a similar notion of centrality.