The advances in convex optimization techniques have offered new formulations of design with improved control over the performance of FIR filters. By using lifting techniques, the design of a length- L FIR filter can be formulated as a convex semidefinite program (SDP) in terms of an L×L matrix that must be rank-1. Although this formulation provides means for introducing highly flexible design constraints on the magnitude and phase responses of the filter, convex solvers implementing interior point methods almost never provide a rank-1 solution matrix. To obtain a rank-1 solution, we propose a novel Directed Iterative Rank Refinement (DIRR) algorithm, where at each iteration a matrix is obtained by solving a convex optimization problem. The semidefinite cost function of that convex optimization problem favors a solution matrix whose dominant singular vector is on a direction determined in the previous iterations. Analytically it is shown that the DIRR iterations provide monotonic improvement, and the global optimum is a fixed point of the iterations. Over a set of design examples it is illustrated that the DIRR requires only a few iterations to converge to an approximately rank-1 solution matrix. The effectiveness of the proposed method and its flexibility are also demonstrated for the cases where in addition to the magnitude constraints, the constraints on the phase and group delay of filter are placed on the designed filter.