Nonnegative matrix factorization (NMF) has an established reputation as a useful data analysis technique in numerous applications. However, its usage in practical situations is undergoing challenges in recent years. The fundamental factor to this is the increasingly growing size of the datasets available and needed in the information sciences. To address this, in this work we propose to use structured random compression, that is, random projections that exploit the data structure, for two NMF variants: classical and separable. In separable NMF (SNMF), the left factors are a subset of the columns of the input matrix. We present suitable formulations for each problem, dealing with different representative algorithms within each one. We show that the resulting compressed techniques are faster than their uncompressed variants, vastly reduce memory demands, and do not encompass any significant deterioration in performance. The proposed structured random projections for SNMF allow to deal with arbitrarily shaped large matrices, beyond the standard limit of tall-and-skinny matrices, granting access to very efficient computations in this general setting. We accompany the algorithmic presentation with theoretical foundations and numerous and diverse examples, showing the suitability of the proposed approaches. Our implementations are publidy available.