Interference alignment (IA) has attracted enormous research interest as it achieves optimal capacity scaling with respect to signal to noise ratio on interference networks. IA has also recently emerged as an effective tool in engineering interference for secrecy protection on wireless wiretap networks. However, despite the numerous works dedicated to IA, two of its fundamental issues, i.e., feasibility conditions and transceiver design, are not completely addressed in the literature. In this two part paper, a generalized interference alignment (GIA) technique is proposed to enhance the IA's capability in secrecy protection. A theoretical framework is established to analyze the two fundamental issues of GIA in Part I and then the performance of GIA in large-scale stochastic networks is characterized to illustrate how GIA benefits secrecy protection in Part II. The theoretical framework for GIA adopts methodologies from algebraic geometry, determines the necessary and sufficient feasibility conditions of GIA, and generates a set of algorithms for solving the GIA problem. This framework sets up a foundation for the development and implementation of GIA.