In this paper, we propose a high-resolution time-frequency rate representation (HR-TFRR) and derive its analytical formula. This formula implies that the HR-TFRR has a narrower frequency rate support than the cubic phase function (CPF). Therefore, the HR-TFRR has a higher frequency rate (FR) resolution than the CPF and can distinguish closer components in the time-frequency rate domain. Due to the bilinear transform, the HR-TFRR suffers the cross term when the instantaneous frequency functions of the components are cross or very close. In order to suppress the cross term, we further propose a smoothed HR-TFRR (SHR-TFRR) by introducing an FR window to the HR-TFRR, which is expressed in the convolution form. Finally, numerical examples demonstrate that the HR-TFRR and the SHR-TFRR outperform their counterparts in frequency rate resolution and cross-term suppression, respectively. In addition, satisfactory performance of the proposed methods in phase parameter estimation and real-world signal analysis is given.