We consider a distributed compressed sensing scenario where many sensors measure correlated sparse signals and the sensors are connected through a network. Correlation between sparse signals is modeled by a partial common support-set. For such a scenario, the main objective of this paper is to develop a greedy pursuit algorithm. We develop a distributed parallel pursuit (dipp) algorithm based on exchange of information about estimated support-sets at sensors. The exchange of information helps to improve estimation of the partial common support-set, that in turn helps to gradually improve estimation of support-sets in all sensors, leading to a better quality reconstruction performance. We provide restricted isometry property (RIP) based theoretical analysis on the algorithm's convergence and reconstruction performance. Under certain theoretical requirements (i.e., under certain assumptions) on the quality of information exchange over the network and RIP parameters of sensor nodes, we show that the dipp algorithm converges to a performance level that depends on a scaled additive measurement noise power (convergence in theory) where the scaling coefficient is a function of RIP parameters and information processing quality parameters. Using simulations, we show practical reconstruction performance of dipp vis-a-vis amount of undersampling, signal-to-measurement-noise ratios and network-connectivity conditions.