Laguerre Gaussian functions serve as a complete and orthonormal basis for a variety of physical problems, such as 2D isotropic quantum harmonic oscillators and circularly symmetric laser modes. In this paper, we propose "discrete Laguerre Gaussian functions," which are defined such that some elegant physical properties are preserved and a fast computation algorithm of complexity O(N logN) is available. Discrete Laguerre Gaussian transforms, as introduced in this paper, inherit nice properties from discrete Laguerre Gaussian functions and admit signal analysis over circularly symmetric patterns. It is demonstrated through examples that discrete Laguerre Gaussian transforms find applications in circular pattern keypoints selection, object detection, image compression, rotational invariance feature for pattern recognition, and rotational angle estimation.