

We study non-uniformly expanding systems on a compact Riemannian manifold admitting critical sets. Under some general conditions, we construct a Gibbs–Markov–Young structure on a disk whose centre's preimages are dense in the manifold. The result has the following application: in a previous study, the authors showed that the decay of correlations implies the existence of tower structure whose return time decays at the same rate. However, for technical reasons, they have to assume that the density function for the absolutely continuous measure is bounded away from 0. Now we remove this constraint and provide the arguments for the more general results.