This paper studies an extension and improvement of the joint detection-decoding algorithm for nonbinary LDPC-coded modulation systems. The iterative joint detection-decoding (IJDD) algorithm in [1] combines nonbinary LDPC decoding with signal detection based on the hard-message passing strategy, resulting in significantly reduced decoding complexity. However, it applies only to majority-logic decodable nonbinary LDPC codes with high column weight. For nonbinary LDPC codes with low column weight, a noticeable performance loss will be incurred. To handle this problem, we propose a reliability-based iterative joint detection-decoding (also termed improved IJDD) algorithm, which combines the accumulated reliability of symbols based on the one-step majority-logic decoding (MLGD) algorithm and a Chase-like local list decoding algorithm. Simulation results show that the improved IJDD algorithm outperforms the IJDD algorithm by about 0.3 dB using nonbinary LDPC codes with high column weight, and by about 3 dB using nonbinary LDPC codes with low column weight ($d_v = 4$), while maintaining the low complexity of decoding. Compared to the FFT-QSPA, the proposed algorithm has a performance degradation of 0.5 dB in the high column weight regime, and about 1 dB in the low column weight regime.