

Our prior work has studied the enforcement of opacity security properties using insertion functions. Given a system that is not opaque, the so-called All Insertion Structure (AIS) is a game structure, played by the system and the insertion function, that embeds all valid insertion functions. In this paper, we first propose a more compact AIS that can be constructed with lower computational complexity. We then introduce the maximum total cost and the maximum mean cost, and use them as quantitative objectives to solve for optimal insertion functions. Specifically, we first determine if an insertion function with a finite total cost exists. If such an insertion function exists, we synthesize an optimal total-cost insertion function. Otherwise, we construct an optimal mean-cost insertion function. In either case, we find an optimal insertion strategy on the AIS, with respect to the corresponding cost objective. The algorithmic procedures are adapted from results developed for minimax games and mean payoff games. The resulting optimal strategy is represented as a subgraph of the AIS that consists of all the system actions and the optimal insertion actions. Finally, we use this subgraph to synthesize an optimal insertion function that is encoded as an I/O automaton.